Rational minimax approximation via adaptive barycentric representations

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Rational approximation is historically a core topic in approximation theory with applications in fields such as computer arithmetic, signal processing and model order reduction. In this talk I will discuss about recent work [1] (inspired by [2]) with my collaborators on designing robust algorithms for computing best (in the L_∞ norm) rational approximations to continuous functions over an interval $[a, b]$.

A core aspect of this work is to consider rational functions (of type (n, n)) in a *barycentric* representation of the form

$$r(x) = \frac{\sum_{k=0}^{n} \dfrac{\alpha_k}{x - t_k}}{\sum_{k=0}^{n} \dfrac{\beta_k}{x - t_k}},$$

where $\{\alpha_k\}, \{\beta_k\}$ are the barycentric coefficients of r and $\{t_k\}$ are called support nodes which can be freely chosen.

We indicate how these $\{t_k\}$ can be taken in an *adaptive*, problem dependent way that greatly reduces the underlying numerical precision needed to obtain accurate results, in comparison to a more common representation of r involving ratios of polynomials represented in monomial or Chebyshev bases. An example of this is the problem of determining type (n, n) best rational approximations to $f(x) = |x|, x \in [-1, 1]$ up to $n = 80$, for which Varga, Ruttan and Carpenter [3] used 200-digit arithmetic, whereas with our approach we get similar results with standard 16-digit floating point arithmetic.

References

[1] S.-I. Filip, Y. Nakatsukasa, L.N. Trefethen, and B. Beckermann, Rational minimax approximation via adaptive barycentric representations. *arXiv preprint arXiv:1705.10132* (2017).

[2] Y. Nakatsukasa, O. Sète, and L. N. Trefethen, The AAA algorithm for rational approximation. Technical report, 2016. submitted to *SIAM J. Sci. Comp.*

[3] R. S. Varga, A. Ruttan, and A. D. Carpenter, Numerical results on best uniform rational approximation of $|x|$ on $[-1, +1]$. *Mathematics of the USSR-Sbornik*, 74(2):271, 1993.