Projection of analytic surfaces

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For some robotic problems we need to represent a singular surface that is the projection of a smooth surface embedded in higher dimension.
In this work, we focus on the problem of computing a triangulation of the projection on $\mathbb{R}^3$ of an analytic surface embedded in $\mathbb{R}^4$.

Based on Transversality theory [3] and Singularity Classification [1, 4], we first recall that, under generic assumptions, the set of singularities of the projected surface are generated by only three types of multi-germs: double points, triple points and cross-caps. Then, we will show that how to characterize those singularities as solutions of three systems of equations which are regular under certain assumptions. Finally, using numerical methods [2, 5, 6], we design an algorithm taking as input an analytic surface and returning a triangulation isotopic to its projected surface.

References

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