A Symbolic Approach for Solving Algebraic Riccati Equations

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A classical and thoroughly studied problem in automatic control theory is the H_∞ control of linear systems. Given a linear dynamical system, the objective is to synthesize controllers that achieve stabilization and guarantee some performance criteria according to the H_∞-norm. This problem, which can be expressed as a mathematical optimization problem, is usually reformulated as a problem of solving nonlinear matrix equations known as *Algebraic Riccati Equations*. Such equations are well-known since they arise in various problems of automatic control and signal processing such as optimal control, Kalman filtering, estimation problems, etc. In the case of the H_∞ control problem, this gives rise to an algebraic Riccati equation of the following form

$$X\,A + A^T\,X - X\,B\,B^T\,X + C^T\,C = 0, \tag{1}$$

where $A, B, C \in \mathbb{R}^{n\times n}$ are constant matrices with real entries and $X \in \mathbb{R}^{n\times n}$ is a symmetric matrix one is seeking for. Under certain conditions on the matrices A, B and C, the goal is then to compute a positive definite matrix X that is solution of (1).

Most of the existing methods for solving (1) rely on purely numerical routines (see [2] and references therein). However, due to large model uncertainties, these methods are too conservative, and unfixed model parameters shall be considered in certain applications, that is a model with matrices A, B and C with unknown parameters.

In this presentation, following the work given in [1], we propose a new computer algebra approach for the study of algebraic Riccati equations that depend on a set of unknown parameters. More precisely, using classical techniques from real algebraic geometry (Gröbner bases, univariate representations and discriminant varieties), we examine the algebraic systems that stem from these equations. The conducted study allows us to exhibit some interesting properties of these systems while it eases the computation of their solutions (see [3] for details).

The presented symbolic approach, which is interesting in the context of adaptive control, is illustrated through a classical example, where explicit formulas are obtained for the robust controller and whose robust margin depends only on the parameters of the systems.

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