A New Approach for Solving the Permutation Code Equivalence Problem

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We study the algorithmic problem of deciding whether two linear codes consist of the same codewords up to a permutation on the codeword coordinates. This problem is called Permutation Code Equivalence. Using simple tools from linear algebra, we show that this problem can be solved for many instances in polynomial time. Our approach starts from a new modeling based on a quadratic multivariate polynomial system, S, describing the solution set of the problem.

However, since our system involves n^2 variables for codes of length n, generic algorithms that solve multivariate systems by computing Gröbner bases become very rapidly impractical.

We consider particular instances where the solving can be accelerated by

1. extracting from the system S linear equations involving only a "block" of n variables $x_1, \ldots, x_n$,
2. computing a Gröbner basis of the system obtained by keeping from S only the previous linear equations and the equations depending only on $x_1, \ldots, x_n$,
3. adding this Gröbner basis to the initial system.

The "block" Gröbner bases are easy to compute, and produce many linear equations that permit to solve the initial system.

This is a joint work with Ayoub Otmani and Mohamed Saeed-Taha.